## Determination of the probability that a planet in a circular orbit around a star will transit the face of its star as seen from Earth. (Robert Douglas)

Let $\mathrm{d}=$ the diameter of the star and $\mathrm{D}=$ the distance of the planet from the star.
The probability is $\mathbf{p}=\mathbf{d} / 2 \mathbf{D}$.


Proof. All the rays of light from the star are parallel as seen from Earth. The planet can only be seen to cross the face of its star as seen from Earth if the planet lies on the blue arc of length $S . \theta$ is the angle in radians subtended by $S . \theta=S / D \sim d / D$ as $S \sim d$. You might at first think that p is equal to the length S divided by $\pi \mathrm{D}$ (the length of a semicircle of radius $D$ ). This would give $p=S / \pi D \sim d / \pi D$. But we must work in three dimensions instead of two dimensions. Probability $p$ is actually the ratio of two areas, not the ratio of two lengths. The first area A is that of a cylinder C (without a top or bottom) gotten by revolving the blue arc around the vertical axis through the center of the Sun. p is A divided by the area of a sphere of radius D. The planet lies on this
sphere and the sphere's area is $4 \pi \mathrm{D}^{2}$. How do we determine A ? C is a cylinder whose vertical side is slightly bulging. But since the angle $\theta$ is very small, the blue arc of length $S$ can be replaced with a vertical line of length d, giving us a normal cylinder (i.e., one with a non-bulging side). The area of any cylinder (excluding its top and bottom) is its circumference times its height. The circumference of C is $2 \pi \mathrm{D}$ and its height is d . So $\mathrm{A}=2 \pi \mathrm{dD}$. Thus $\mathrm{p}=\mathrm{A} / 4 \pi \mathrm{D}^{2}=\mathrm{d} / 2 \mathrm{D}$, the desired result.

Example 1. Let us ask for the value of p when the planet's distance from its star is the same as the distance of Earth from the Sun, and when the Star's diameter is that of the Sun. Here $\mathrm{D}=150$ million kms and $\mathrm{d}=1.3927$ million km. Thus $\mathrm{p}=0.0046$ or $0.46 \%$.

## Then we can ask the question:

For each planet that we find transiting a star, using the above values for d and D , how many other planets, having the same values for d and D , can we expect to have orbits tilted so that no transit occurs as seen from Earth?

If $\mathrm{N}=$ the number of planets not detected for every one that is detected, then

$$
\mathrm{N} / 1=(1-\mathrm{p}) / \mathrm{p}=1 / \mathrm{p}-1
$$

(the ratio of the probability of not finding a planet transiting its star to the probability of finding it transiting its star).

Using the values in example 1 gives $\mathrm{N}=216$ planets that cannot be detected using the transit procedure.

Example 2. Let's use Mercury as the planet, again with the star being the Sun. Here dis again, of course, 1.3927 million kms. But Mercury has an orbit that is far from being circular. At its closest point to the Sun it is 29 million km away. At its farthest it is 47 million km away. If we assume a circular orbit for our planet using $\mathrm{D}=29$ million, then $\mathrm{p}=2.4 \%$ and $\mathrm{N}=41$. Using $\mathrm{D}=47$ million, $\mathrm{p}=1.48 \%$ and $\mathrm{N}=66.6$.

Many of the planets detected using the transit procedure are much closer to their star than 29 million km . This would reduce the value of N .

Finally, I note that it isn't the actual values of d and D that matter, it is their ratio $\mathrm{d} / \mathrm{D}$. For different values $d^{\prime}$ and $D^{\prime}$, we can have $d^{\prime} / D^{\prime}=d / D$.

Reference. https://physics.stackexchange.com/questions/26420/what-percent-of-planets-are-in-the-position-that-they-could-be-viewed-edge-on-fr
I wish to thank Steve Gottlieb for bringing this article to my attention.

